Federated Self-supervised Learning for Heterogeneous Clients

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Abstract

Federated Learning has become an important learning paradigm due to its privacy and computational benefits. As the field advances, two key challenges that still remain to be addressed are: (1) system heterogeneity - variability in the compute and/or data resources present on each client, and (2) lack of labeled data in certain federated settings. Several recent developments have tried to overcome these challenges independently. In this work, we propose a unified and systematic framework, *Heterogeneous Self-supervised Federated Learning* (Hetero-SSFL) for enabling self-supervised learning with federation on heterogeneous clients. The proposed framework allows collaborative representation learning across all the clients without imposing architectural constraints or requiring presence of labeled data. The key idea in Hetero-SSFL is to let each client train its unique self-supervised model and enable the joint learning across clients by aligning the lower dimensional representations on a common dataset. The entire training procedure could be viewed as self and peer-supervised as both the local training and the alignment procedures do not require presence of any labeled data. As in conventional self-supervised learning, the obtained client models are task independent and can be used for varied end-tasks. We provide a convergence guarantee of the proposed framework for non-convex objectives in heterogeneous settings and also empirically demonstrate that our proposed approach outperforms the state of the art methods by a significant margin.

1 Introduction

Federated learning has become an important learning paradigm for training algorithms in a privacy preserving way and has gained a lot of interest in the recent past. While traditional federated learning is capable of learning high performing models [Li et al., 2021b], two practical challenges that remain under studied are: system heterogeneity, and lack of labeled data. In several real world scenarios, the clients involved in the training process are highly heterogeneous in terms of their data and compute resources. Requiring each client to train identical models like in traditional FL may thus be severely limiting. Similarly, assuming each client's local data resources to be fully labeled may not be pragmatic as annotating data is time consuming and may require expertise.

Our focus in this work is to jointly address these two challenges in a systemic way so as to make FL more practical. Some prior works have studied these two issues separately. For instance, to help with scarcity of labeled data on local clients, use of semi-supervised learning has been recently studied [Zhang et al., 2021b, Jeong et al., 2021, Lu et al., 2022]; other very recently proposed approaches suggest the use of locally self-supervised learning [Zhuang et al., 2021, 2022, He et al., 2022] assuming identical model architectures on each client. System heterogeneity on the other hand is commonly addressed by building heterogeneous personalised models on clients as suggested

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in [Makhija et al., 2022, Tan et al., 2022, Jiang et al., 2020, Fallah et al., 2020] but all of these approaches assume the presence of fully labeled data on all of the clients.

Though the above mentioned approaches have shown promising results in dealing with both the problems in isolation, in certain real-world applications these two challenges need to be jointly addressed in a holistic way. For example, some of the key applications of FL in cross-silo settings are in healthcare systems where different hospitals may possess varying amounts of private medical images or in cross-device settings where end-devices such as sensors acquire loads of unlabeled data. In both these cases, the data can neither be centralised nor can undergo extensive annotations, moreover to expect each client (a hospital or a sensor in the above examples) to train local models of identical capacities will be highly inhibiting.

In this work, we circumvent these problems by proposing a new framework that allows clients to train self-supervised models with unique (locally tuned) structures while still using the learnings from other clients in the network, in an architecture agnostic way. To achieve this, we add a proximal term to each client's loss function that helps in aligning the learnt lower dimensional representations (aka embeddings) across clients. This way of transferring and acquiring global knowledge allows variability in client model architectures while keeping the entire learning process independent of labeled training data. Furthermore, we use a kernel based distance metric for proximity calculation which provides much more flexibility to the clients in defining their own lower dimensional space without any constraints. To the best of our knowledge, this is the first work that studies collaborative unsupervised learning in a distributed heterogeneous client setting.

Our Contributions are summarized as follows :

1. Our main contribution is the new framework, *Hetero-SSFL*, for training heterogeneous models in a federated setting in an unsupervised way. Hetero-SSFL allows each client to train its own customized model architecture, using local data and computing resources while also utilising unlabeled supervision from peers.
2. We perform extensive experimentation to provide a thorough evaluation of the approach in various kinds of practical settings. We observe that the proposed flexible approach substantially improves the accuracy of unsupervised federated learning in heterogeneous settings.
3. We also provide theoretical analysis and convergence guarantee of the algorithm for non-convex loss functions.

Organization. The rest of the paper is organised as follows. Section 2 provides a brief background on Federated Learning, Self-supervised Learning and related developments. In Section 3, we go over the preliminaries and then propose our framework. We study the convergence guarantee of the algorithm in Section 4 and include the related proofs in the Appendix. A thorough experimental evaluation of our method on different types of datasets is presented in Section 5 and we conclude the paper in Section 6.

2 Related Work

This section provides an overview of the most relevant prior work in the fields of federated learning, self-supervised learning and federated self-supervised learning.

2.1 Federated Learning(FL)

The problem of training machine learning models in distributed environments with restrictions on data/model sharing was studied by several researchers in the data mining community in the early 2000s under titles such as (privacy preserving) distributed data mining [Kargupta and Park, 2000, Gan et al., 2017, Aggarwal and Yu, 2008]. Much of this work was for specific procedures such as distributed clustering [Merugu and Ghosh, Nov, 2003] including in heterogenous settings with different feature spaces [Merugu and Ghosh, 2005], and distributed PCA [Kargupta et al., 2001], or for specific models such as SVMs [Yu et al., 2006]. Subsequently, with the re-surfacing in popularity of neural networks and proliferation of powerful deep learning approaches, the term "Federated Learning" got coined and popularized largely by an influential paper [McMahan et al., 2017] which introduced FedAvg. Indeed FedAvg is now considered the standard distributed training method for Federated Learning, which has become a very active area of research since then. One of the key challenges in this setting is the presence of non-iid datasets across clients. Several modifications of the original FedAvg algorithm have been proposed to address this challenge. Some of these approaches focus on finding better solutions to the optimization problem to prevent the divergence of the global solution [Li et al., 2020, Karimireddy et al., 2020, Zhang et al., 2021a, Pathak and Wainwright, 2020, Acar et al., 2021] whereas some suggest the modification of the training procedure to incorporate appropriate aggregation of the local models [Chen and Chao, 2021, Wang et al., 2020a, Yurochkin et al., 2019, Wang et al., 2020b, Singh and Jaggi, 2020]. Creating local personalised models for each client to improve the overall method is also well researched [Collins et al., 2021, Ghosh et al., 2020, Smith et al., 2017a, Li et al., 2021b, Smith et al., 2017b, Sattler et al., 2021, Yu et al., 2020]. Also, augmenting the data distributions by creating additional data instances for learning [Hao et al., 2021, Goetz and Tewari, 2020, Luo et al., 2021] has seen to help with the collaborative effort in federated learning.

2.2 Self-supervised learning(SSL)

SSL aims at training high performing complex networks without the requirement of large labeled datasets. It does not require explicitly labeled data and target pairs but instead uses inherent structure in the data for representation learning. The two types of SSL methods include contrastive based and non-contrastive based SSL. The contrastive methods work on triplets of the form (x, x', y) where x and x' are different views of the same instance and y is a non-compatible data instance. The key idea is to train models by adjusting the parameters in such a way that the representations(embeddings) obtained for x and x' are nearly identical and that of x and y are unalike. Since obtaining these triplets from the dataset does not necessarily require presence of the labels, for example for images, x and x' could be different augmentations of the same image and y could be an image of a different object, these methods could learn from large volume of data. Recent works in this field suggest many different ways of generating the data triplets for training [Chen et al., 2020a, He et al., 2020, Wu et al., 2018, Yan et al., 2020]. Non-contrastive methods, on the other hand, work without generating explicit triplets. One form of the non-contrastive methods use clustering based methods to form a group of objects and use the cluster assignments as pseudo-labels for training the model [Caron et al., 2018, 2020]. Another form of non-contrastive methods use joint embedding architectures like Siamese network but keep one of the architectures to provide target embeddings for training [Chen and He, 2020, Grill et al., 2020, Chen et al., 2020b].

2.3 Federated Self-supervised Learning

The interest in self-supervised federated learning is relatively new. Although MOON [Li et al., 2021a] proposed using a contrastive loss in supervised federated learning, it was FedU [Zhuang et al., 2021] that proposed a way to leverage unlabeled data in a decentralised setting. Subsequently, two parallel works [Zhuang et al., 2022], [He et al., 2022] proposed frameworks for training self-supervised models in a federated setting and compared different self-supervised models. In all of these approaches though, the clients train local models of identical architectures. This common architecture is communicated by the server at the beginning of the training. In each round, the clients perform update steps on the local models and send the local models to the central server at the end of the round. The server appropriately aggregates the client models to generate the global model for that round and communicates the global model parameters back to each client. The prominent idea in these latest developments is the divergence aware updates of the local model to the global model wherein, if the local models are very different from the global model, the local models are not updated with the global model, which can be seen as an initial step towards creating more personalised models for each client.

3 Methodology

In order to enable federated learning for heterogeneous clients in an unsupervised way, we propose a new framework using self-supervised federated learning and name it Hetero-SSFL. This framework is unique as it allows heterogeneous clients to participate in the training process and also exhibits high empirical performance. The convergence guarantee of the algorithm for non-convex loss functions is also shown. In this section we first formally describe the problem setting and then elaborate on our proposed solution.

3.1 Problem Definition

A federated learning setting consists of a set of clients $k \in |N|$, with each client k having local data instances $\mathcal{X}_k$ of size n_k drawn from the local data distribution $\boldsymbol{D}_k$. The distributions $\boldsymbol{D}_k$ of different clients are more likely to be non-identical, for example, each client may have access to objects of only a few categories or classes different from other clients' object categories. The local objective at k^{th} client is to solve for $\min_{\mathcal{W}_k} \mathcal{L}_k = \ell(\mathcal{W}_k)$ where $\ell(.)$ is some loss function. After the local optimization, the federation part of the learning allows the server to access all the clients' learnings in the form of the learned set of weights $\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_N$ and utilise it for achieving global objective. In traditional FL methods, it is assumed that all $\mathcal{W}_k$'s are identical in shape and in each round the server combines the local models to learn a global model $\hat{\mathcal{W}}$ of the form $\hat{\mathcal{W}} = g(\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_N)$, where g is an appropriate aggregation function, and sends $\hat{\mathcal{W}}$ back to the clients for further updates. The novelty of our solution is in the fact that our solution does not require the local models (and thus the local weights) to be of identical size. The way we utilise the global knowledge, $\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_N$, to achieve collaboration for each client's benefit, is also different.

3.2 Hetero-SSFL Framework

We introduce the Hetero-SSFL framework in this section and provide details of all the components in the framework here. The end-to-end pipeline is depicted in Figure 1 and the algorithm is detailed in Algorithm 1.

Local Training We assume each client contains unlabeled datasets and employ self-supervised learning on each client. While there are many ways to achieve self-supervision, in this work we focus on non-contrastive methods like BYOL [Grill et al., 2020] and SimSiam [Chen and He, 2020] which involve two embedding architectures like Siamese networks for achieving self-supervision. The two parallel networks are called the *online* network and *target* network respectively. The target network's task is to provide targets for regression to the online network whereas the online network is trained to learn representations. The target network consists of an encoder and the online network is composed of an encoder and a prediction layer on top of it, whose role is to learn better representations under self-supervision loss. The weights of the target network are just a moving average of the weights of the online network and the target network can thus be thought of as a stable version of the online network. In federated self-supervised learning, the local set of parameters for each client k include $\mathcal{W}_k = \{W_k^o, W_k^p, W_k^t\}$ where W_k^o parameterises the representation learning component of the online network with W_k^p being the prediction layer and W_k^t is the target network. The local loss at each client $\ell(.)$ is similar to self-supervised loss used in central self-supervised learning and is given by -

$$\ell(\mathcal{W}_k) = ||\mathcal{F}(v'; [W_k^o, W_k^p]) - \mathcal{F}'(v''; W_k^t)||^2,$$

i.e., it is the mean squared error between the outputs of the online network and the target network on different views(augmentations), v' and v'', of the same image v. $\mathcal{F}(,;.)$ and $\mathcal{F}'(,;.)$ denote the online and target network functions respectively.

In our framework, the learning on each client is also guided by peers to bring about collaboration. We use this guidance in the form of representations that each client locally learns for a common set of instances. The intuition is that the clients can work together to uncover the hidden representation structure in the data in an unsupervised way. Specifically, we modify the local loss function at each client to contain a proximal term that measures the distance between the local representations and the representations obtained on all other clients. The local loss function at each client thus becomes -

$$\min_{\mathcal{W}_k} \mathcal{L}_k = \ell(\mathcal{W}_k) + \mu \mathrm{d}\left(\Phi_i(\mathbf{X}; \mathcal{W}_i^o), \bar{\Phi}(\mathbf{X}; (t-1))\right)$$

where $\mathrm{d}(.,.)$ is a suitable distance metric, and $\bar{\Phi}$ denotes the aggregated representations of other clients

$$\bar{\Phi}(\mathbf{X}; (t)) = \sum_{j=1}^{N} w_j \Phi_j(\mathbf{X}; \mathcal{W}_j^o(t)).$$

Here, w_j is the weight given to the j^{th} client and could be kept higher for clients with larger resources or learnt in the training pipeline. The representations $\Phi(\mathbf{X};.)$ are obtained from the output of the client's online network under parameters $\mathcal{W}^o = [W^o, W^p]$ at a layer suitable for defining the representation space, and this could potentially be different from $\mathcal{F}$ which denotes the entire online network. In our framework, we use the output of the predictor layer for an instance x as $\Phi(x;.)$.

Communication with the Server Different from traditional FL where each client sends the local models to server and the server creates a global model by performing element-wise aggregation on the local model weights, we gather representations at the server. We assume that the server has access to an unlabeled dataset(which could be easily obtained in practice from open sources) and call it *Representation Alignment Dataset*(RAD). In each global training round t, server sends the RAD to all the clients. All the clients use the RAD, **X**, in the loss function as described above while simultaneously training local models. After the clients finish local epochs, the final representations obtained for RAD **X** on each client, $\Phi_k(\mathbf{X}; \mathcal{W}_k^o)$, are sent to the server. The server then aggregates the local representations from all clients to form the global representation matrix $\bar{\Phi}(\mathbf{X}; (t))$ which is then again sent to the clients for training in the next round.

3.3 The Proximal Term

In this sub-section we describe the second part of the loss function and provide details on the proximal term and the distance function.

The role of the proximal term is to compare the representations obtained from various neural networks. This requires the distance function to be able to meaningfully capture the similarity in the high dimensional representational space. Moreover, the distance function should be such that it is sensitive to the changes that affect functional behavior and does not change as much with the inconsequential modifications like change in random initializations. The types of distance functions used in the literature for comparing neural network representations include Canonical Correlation Analysis(CCA), Centered Kernel ALignment(CKA) and Procustres distance based measures [Ding et al., 2021, Kornblith et al., 2019]. While the Procustres distance based metrics cannot compare the representations obtained in spaces of different dimensions, the CCA based metrics are not suitable for training through backpropagation. In contrast, properties of the CKA metric like ability to learn similarities between layers of different widths, invariance to random initializations, invertible linear transformations and orthogonal transformations lend it useful for use in the proximal term.

CKA measures the distance between the objects in different representation spaces by creating similarity(kernel) matrices in the individual space and then comparing these similarity matrices. The idea is to use the representational similarity matrices to characterize the representation space. CKA takes the activation matrices obtained from the network as inputs and gives a similarity score between 0 and 1 by comparing the similarities between all objects. Specifically, if $A_i \in \mathbb{R}^{L \times d_i}$ and $A_j \in \mathbb{R}^{L \times d_j}$ are the activation(representation) matrices for clients i and j for the RAD **X** of size L, the distance between A_i and A_j using Linear-CKA is computed as -

$$\text{Linear CKA}(A_i A_i^T, A_j A_j^T) = \text{Linear CKA}(K_i, K_j) = \frac{||A_j^T A_i||_F^2}{||A_i^T A_i||_F ||A_j^T A_j||_F}, \tag{1}$$

where K_i and K_j are kernel matrices for any choice of kernel $\mathcal{K}$ such that we have $K_i(p, q) = \mathcal{K}(A_i(p, :), A_i(q, :))$. CKA also allows choice of other kernels like RBF-kernel, polynomial kernel etc. but we observed that the results with Linear-CKA are as good as those with RBF kernel as also reported in [Kornblith et al., 2019]. Thus we focus only on Linear-CKA here.

Finally, the local loss at each client becomes

$$\min_{\mathcal{W}_k} \mathcal{L}_k = \ell(\mathcal{W}_k) + \mu \,\text{Linear-CKA}\Big(K_k, \bar{K}(t-1)\Big), \tag{2}$$

with $\bar{K}(t-1) = \sum_{j=1}^{N} w_j K_j(t-1)$. When $\mu = 0$, each client is training its own local models without any peer-supervision and when $\mu \to \infty$ the local models try to converge on the RAD representations without caring about the SSL loss. In such cases, if the size of RAD is increased arbitrarily and models have similar architectures, the local models become identical.

4 Convergence Analysis

In this section we provide insights on convergence of the proposed framework. The convergence guarantee is shown to hold under mild assumptions which are commonly made in the literature [Wang et al., 2020b, Tan et al., 2022].

Assumption 4.1 (Lipschitz Smoothness)**.** The local loss function on every client k, $\mathcal{L}_k$, is assumed to be L_1-Lipschitz smooth, which implies:

$$\mathcal{L}_k(a) - \mathcal{L}_k(b) \leq \nabla \mathcal{L}_k(b)^\top (\mathcal{W}_k(a) - \mathcal{W}_k(b)) + \frac{L_1}{2}||\mathcal{W}_k(a) - \mathcal{W}_k(b)||_2^2, \quad \forall\ a, b. \tag{3}$$

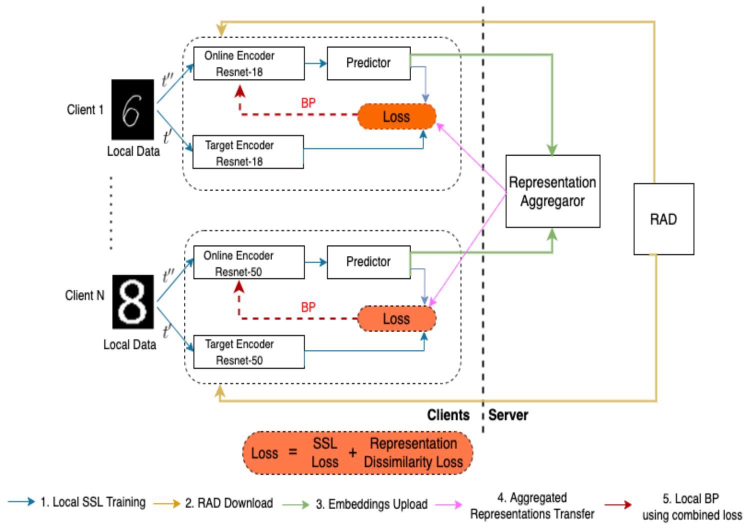

Figure 1: Overview of the proposed framework: Hetero-SSFL. In each training round, the server sends the RAD and the aggregated representation matrix to all clients, each client trains local self-supervised models by optimising the combined loss, the representations obtained using local model on RAD are sent back to the server which aggregates the representations and sends it back to all clients for next training round. Note: the local models are named Resnet-18 and Resnet-50 as examples to depict heterogeneous client models.

Assumption 4.2 (Stochastic Gradient)**.** The local stochastic gradient for each client k, at any time t, $g_{k,t}$ is an unbiased estimator of the gradient, has bounded variance and the expectation of its norm is bounded above. That is, we have

$$\mathbb{E}[g_{k,t}] = \nabla \mathcal{L}_k \quad \text{and} \quad \text{Var}(g_{k,t}) \leq \sigma^2,$$
$$\mathbb{E}[||g_{k,t}||_2] \leq P.$$

Assumption 4.3 (Representation Norm)**.** The norm of the representations obtained for any instance on any client is bounded above by R. That is:

$$||\Phi_k(x; \mathcal{W}_k)||_2 \leq R.$$

Assumption 4.4 (Lipschitz Continuity)**.** The embedding function $\Phi(.;.)$ for all clients is L_2-Lipschitz continuous, which implies:

$$||\Phi_k(.; \mathcal{W}_a) - \Phi_k(.; \mathcal{W}_b)||_2 \leq L_2||\mathcal{W}_a - \mathcal{W}_b||_2.$$

We start with the following lemma on the reduction of the local loss after E local epochs.

Lemma 4.5. *(Local Epochs) In each communication round, the local loss $\mathcal{L}$ for each client reduces after E local epochs and is bounded as below under the Assumption 4.2.*

$$\mathbb{E}[\mathcal{L}_E] \leq \mathcal{L}_0 - (\eta - \frac{L_1\eta^2}{2}) \sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 + \frac{L_1 E \eta^2}{2}\sigma^2. \tag{4}$$

where $\mathcal{L}_0$ and $\mathcal{L}_E$ denote the loss before and after E local epochs respectively and η is learning rate.

Proof of Lemma 4.5 is in Appendix. Lemma 4.5 shows the bound on the client's local loss after the completion of local epochs under one global communication round.

After the local training, the representations from all clients are sent to the server. Our next lemma 4.6 shows a bound on the expected loss after every global representation update at the server.

Lemma 4.6. *(Representation Update) In each global communication round, for each client, after the E local updates, the global representation matrix is updated at the server and the loss function for any client k gets modified to $\mathcal{L}_{E'}$ from $\mathcal{L}_E$ and could be bounded as -*

$$\mathbb{E}[\mathcal{L}_{E'}] \leq \mathbb{E}[\mathcal{L}_E] + 2\mu\eta L_2 P R^3 L^2. \tag{5}$$

Algorithm 1 Hetero-SSFL Algorithm for Heterogeneous clients

Input: number of clients N, number of global communication rounds T, number of local epochs E, parameter μ, weight vector for clients $[w_1, w_2, \dots w_n]$
Output: Final set of personalised models $\mathcal{W}_1(T), \mathcal{W}_2(T) \dots \mathcal{W}_n(T)$
At Server -
Initialize $\mathcal{W}_1(0), \mathcal{W}_2(0) \dots \mathcal{W}_N(0)$
for $t = 1$ **to** T **do**
  RAD = **X**
  for each client j **do**
    $A_j(\mathbf{X}) = \Phi_j(\mathbf{X}; \mathcal{W}_j(t-1))$
    $K_j = \mathcal{K}(A_j(\mathbf{X}), A_j(\mathbf{X}))$
  end for
  $\bar{K}(t-1) = \sum_{j=1}^{N} w_j K_j$
  Select a subset of clients $\mathcal{N}_t$
  for each selected client $i \in \mathcal{N}_t$ **do**
    $\mathcal{W}_i(t) = \textbf{LocalTraining}(\mathcal{W}_i(t-1), \bar{K}(t-1), \mathbf{X}, \mu)$
  end for
end for
Return $\mathcal{W}_1(T), \mathcal{W}_2(T) \dots \mathcal{W}_n(T)$
LocalTraining$(\mathcal{W}_i(t-1), \bar{K}(t-1), \mathbf{X}, \mu)$
Initialize $\mathcal{W}_i(t)$ with $(\mathcal{W}_i(t-1))$
for each local epoch **do**
  $A_i(\mathbf{X}) = \Phi_i(\mathbf{X}; \mathcal{W}_i(t))$
  $K_i = \mathcal{K}(A_i(\mathbf{X}), A_i(\mathbf{X}))$
  Update $\mathcal{W}_i(t)$ using SGD for loss in equation (2)
end for
Return $\mathcal{W}_i(t)$

Proof of Lemma 4.6 is in Appendix.

Equipped with the results from Lemmas 4.5 and 4.6, we are now ready to state our main result which provides a total deviation bound for an entire training round. Theorem 4.7 shows the divergence in loss for any client after completion of one round. We can guarantee the convergence by appropriately choosing μ and η which leads to a certain expected decrease in the loss function. The result holds for convex as well as non-convex loss functions.

Theorem 4.7 (Convergence). *After one global round, the loss function of any client decreases and is bounded as shown below:*

$$\mathbb{E}[\mathcal{L}_{E'}] \leq \mathcal{L}_0 - (\eta - \frac{L_1\eta^2}{2}) \sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 + \frac{L_1 E \eta^2}{2}\sigma^2 + 2\mu\eta L_2 P R^3 L^2. \tag{6}$$

Thus, if we choose η and μ such that

$$\eta < \frac{2(\sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 - 2\mu L_2 P R^3 L^2)}{L_1(\sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 + E\sigma^2)}, \quad \mu < \frac{\sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2}{2 L_2 P R^3 L^2},$$

then the convergence of Algorithm 1 is guaranteed.

The proof of Theorem 4.7 follows directly from the results of Lemmas 4.5 and 4.6.

5 Experiments

In this section we describe the experimental setup and present results for our method alongside the popular baselines. We simulate system heterogeneity by randomly choosing different architectures for local client models. Within system heterogeneity we evaluate our model under varying statistical settings by manipulating the data distributions across clients to be IID or non-IID. We also demonstrate the performance of our framework with increase in number of clients.

5.1 Experimental Details

Datasets We test our framework on image classification task and use three datasets to compare against the baselines: CIFAR-10, CIFAR-100 and Tiny-Imagenet as suggested by several works in FL and the popular federated learning benchmark LEAF [Caldas et al., 2019]. CIFAR-10 and CIFAR-100 contain 50,000 train and 10,000 test colored images for 10 classes and 100 classes respectively and Tiny-Imagenet has 100,000 images for 200 classes.

Implementation Details For the FL simulations, we explore two types of settings, non-IID setting and IID setting. In the IID-setting, we assume each client to have access to the data of all classes and in the non-IID setting we assume each client has access to data of only a few disjoint classes. For example, for CIFAR-10 client 1 might have access to objects of classes $\{1, 2\}$ versus client 2 with access to $\{3, 4\}$ and likewise. The IID setting is created by dividing all the instances between the clients. For the non-IID setting, for each client a fraction of classes is sampled and then instances are divided amongst the clients containing specific classes. In the cases when number of clients, N, is less than total number of classes K, each client is assumed to have access to all instances corresponding to $\frac{K}{N}$ classes. The choice of encoders at each client is data and application dependent. Since we are dealing with images, to create heterogeneous clients we use Resnet-18 and Resnet-34 models as encoders for our method. Since all the baselines work in homogeneous client settings and hence have to use the same model, we use Resnet-34 at each client in the baselines for comparison. The total number of global communication rounds is kept to 200 for all methods with each client performing $E = 5$ local epochs in each round. We use SGD with momentum $= 0.9$, batch size $= 200$ and learning rate $= .032$ for training all models. The weight vector used in aggregating the representations **w** is set to be uniform to $\frac{1}{n}$. All these models are trained on a machine with 4 GeForce RTX 3090 GPUs with each GPU having about 24gb of memory.

Baselines We compare our method against two very recently proposed federated self-supervised baselines: FedU and FedEMA, which reflect the current state-of-the-art. Since the source code for these baselines is not publicly available, (though they have been promised in the future as per our personal communications), we perform these comparisons using our implementation and with the best parameters reported in these papers. We also create additional baselines by combining central SSL algorithms like BYOL and SimSiam with the federated learning procedure of FedAvg to make FedBYOL and FedSimSiam. The performance of supervised FedAvg and centralised BYOL is also reported for comparisons as theoretical baselines. All of these methods however work in homogeneous settings, hence we compare the homogeneous setting of baselines with Resnet-34 architecture with our heterogeneous setting having a mix of Resnet-34 and Resnet-18 architectures. One additional baseline is the SSFL algorithm, but due to the unavailability of the source code and lack of precise implementation details in the paper, we could not compare against it.

Evaluation We follow the linear evaluation protocol as mentioned in [Zhuang et al., 2021] to test the models. Under this protocol, the local encoders are trained in self-supervised way without any labeled data and then a linear classifier is trained on top of the encoder after freezing the encoder parameters. We use a linear fully connected layer with 1000 neurons as the classifier and train it for 100 epochs with Adam optimizer, batch size = 512 and learning rate = 0.003. The reported results are the accuracy of the linear classifier on a separate test dataset.

Other parameters The other parameters important for our approach are μ which controls the weight of the proximal term in the loss function, and the size of the RAD dataset. We tune the hyperparameter μ using a validation set and found the best values of μ to be 0.5 for the CIFAR datasets and 1 for the Tiny-Imagenet dataset. For the RAD size we observe that increasing the size of RAD increases performance but has an effect on the speed of simulation so we fix that size to be 5000 in our experiments. On studying the effect of varying number of local epochs we observed that unlike the FedAvg based algorithms our method is stable with increasing local epochs.

5.2 Results

We show the performance of our method in comparison with baselines in non-IID settings in Table 1 and that for IID settings in Table 2. We observe that our method outperforms the baselines by a significant margin and is slightly worse than the central BYOL which implies that we are able to achieve successful collaboration amongst clients. We also report the performance of our method and high-performing baselines with increase in scale in Table 3. As the number of clients increase, the assumption on high availability of clients is relaxed and in every round only a fraction of clients

Table 1: Test Accuracy comparison of our method with baselines under the linear evaluation protocol in non-IID settings.

Method	CIFAR-10	CIFAR-100	Tiny-Imagenet
FedBYoL	78.1 ± 1.1	58.1 ± 0.3	31.8 ± 2.0
FedSimSiam	76.7 ± 1.5	53.2 ± 0.16	29.1 ± 1.2
FedU	79.6 ± 0.5	58.9 ± 0.12	58.23 ± 2.3
FedEMA	81.2 ± 1.6	61.8 ± 0.31	58.2 ± 3.1
Hetero-SSFL	$\mathbf{90.29 \pm 0.3}$	$\mathbf{67.7 \pm 0.21}$	$\mathbf{61.5 \pm 1.8}$
Supervised FedAvg	68.5 ± 1.7	44.29 ± 1.34	27.2 ± 3.2
Central-BYOL (IID upper bound)	94.3 ± 0.2	74.2 ± 0.67	78.7 ± 1.3

Table 2: Test Accuracy comparison of our method with baselines under the linear evaluation protocol in IID settings(with equal number of classes on all clients).

Dataset	FedBYoL	FedSimSiam	FedU	FedEMA	Hetero-SSFL
CIFAR-10	82.24 ± 0.8	80.4 ± 1.2	81.6 ± 1.8	85.9 ± 0.3	$\mathbf{88.5 \pm 1.3}$
CIFAR-100	23.8 ± 0.14	22.19 ± 1.8	20.4 ± 0.91	25.7 ± 0.4	$\mathbf{34.8 \pm 0.7}$

are sampled to perform local training, total 20 clients with 5 clients selected per round is denoted as 20clients(5) in Table 3. For additional insights, we show the average change in test accuracy of clients with different architectures when performing standalone training versus collaborative learning in the heterogeneous setting.

Table 3: Test Accuracy of various federated self-supervised methods with change in scale on CIFAR-100 dataset.

Dataset	5clients(5)	20clients(5)	100clients(10)
FedU	58.9 ± 0.12	48.29 ± 1.8	41.1 ± 2.3
FedEMA	61.8 ± 0.31	52.5 ± 0.8	42.0 ± 2.7
Hetero-SSFL	67.7 ± 0.21	57.5 ± 0.9	43.3 ± 1.9

Table 4: Average effect of collaboration on different clients. Column 2 depicts the test accuracy of clients with local models and column 3 the accuracy under Hetero-SSFL.

Client Model Architecture	Local SSL	Hetero-SSFL
Resnet-18	71.9	85.4
Resnet-34	77.8	93.7

6 Conclusion

In this work we study the problem of self-supervised federated learning with heterogeneous clients and propose a novel framework to enable collaboration in such settings. The framework allows clients with varying data/compute resources to partake in the collaborative training procedure. This development enables several practical settings to still gainfully employ self-supervised FL even though the different clients, for example different edge-devices or distinct organisations, might not have similar compute and data resources. The high performance of the framework across multiple datasets indicates its promise in achieving collaboration while the theoretical results guarantee the convergence of the algorithm for fairly general loss functions. In future work we plan to investigate the use of attention based mechanisms for adaptive weighting of different clients to generate aggregate representations based on each client's relative importance. Also, rather than assuming that the server has access to an unlabeled dataset and transmitting RAD from the server to clients, we can create a common generative model in a federated way like federated GANs to generate data instances required for RAD.

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Supplement for "Federated Self-supervised Learning for Heterogeneous Clients"

In this supplementary material, we provide proofs for the key results in the paper in Appendix A. Then, we provide additional experiments in Appendix B.

A Proofs

We first provide proof of Lemma 4.5 in Appendix A.1. Then, the proof of Lemma 4.6 is given in Appendix A.2.

A.1 Proof of Lemma 4.5

Let $\mathcal{L}_E$ denote the loss function after E local epochs. Then, from the Lipschitz smoothness assumption in Assumption 4.1, we obtain that

$$\mathcal{L}_1 \leq \mathcal{L}_0 - \eta[\nabla \mathcal{L}_0^T g_0] + \frac{L_1 \eta^2}{2}[||g_0||^2].$$

Therefore, we have

$$\begin{aligned}
\mathbb{E}[\mathcal{L}_1] &\leq \mathbb{E}[\mathcal{L}_0] - \eta \mathbb{E}[\nabla \mathcal{L}_0^T g_0] + \frac{L_1 \eta^2}{2}\mathbb{E}[||g_0||^2] \\
&= \mathbb{E}[\mathcal{L}_0] - \eta ||\nabla \mathcal{L}_0||^2 + \frac{L_1 \eta^2}{2}\mathbb{E}[||g_0||^2] \\
&= \mathbb{E}[\mathcal{L}_0] - \eta ||\nabla \mathcal{L}_0||^2 + \frac{L_1 \eta^2}{2}(Var(g_0) + \mathbb{E}[||g_0||]^2) \\
&\leq \mathbb{E}[\mathcal{L}_0] - (\eta - \frac{L_1 \eta^2}{2})||\nabla \mathcal{L}_0||^2 + \frac{L_1 \eta^2}{2}\sigma^2.
\end{aligned}$$

By summing the above bounds over E number of local epochs, we find that

$$\begin{aligned}
\sum_{i=1}^{E} \mathbb{E}[\mathcal{L}_i] \leq \sum_{i=0}^{E-1} \mathbb{E}[\mathcal{L}_i] - (\eta - \frac{L_1 \eta^2}{2}) \sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 + \frac{L_1 E \eta^2}{2}\sigma^2 \\
\mathbb{E}[\mathcal{L}_E] \leq \mathcal{L}_0 - (\eta - \frac{L_1 \eta^2}{2}) \sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 + \frac{L_1 E \eta^2}{2}\sigma^2.
\end{aligned}$$

Therefore, if $\eta < \dfrac{2(\sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2)}{L_1(\sum_{i=0}^{E-1} ||\nabla \mathcal{L}_i||^2 + E\sigma^2)}$, we obtain $\mathcal{L}_E \leq \mathcal{L}_0$ and the local loss reduces after E local epochs. As a consequence, we obtain the conclusion of the lemma.

A.2 Proof of Lemma 4.6

For any arbitrary client after the global representation update step, if the loss function gets modified to $\mathcal{L}_{E'}$ from $\mathcal{L}_E$, we have

$$\begin{aligned}
\mathcal{L}_{E'} &= \mathcal{L}_E + \mathcal{L}_{E'} - \mathcal{L}_E, \\
&= \mathcal{L}_E + \mu\Big(\mathrm{d}(.;t) - \mathrm{d}(.;t-1)\Big).
\end{aligned}$$

As for client i, $\mathrm{d}(.;t) = \text{Linear-CKA}(K_i(t), \bar{K}(t))$, we have,

$$\mathcal{L}_{E'} = \mathcal{L}_E + \mu\Big(\text{Linear-CKA}(K_i(t), \bar{K}(t)) - \text{Linear-CKA}(K_i(t), \bar{K}(t-1))\Big).$$

Since this holds for all clients, we drop the client index i going forward and obtain that

$$
\begin{aligned}
\mathcal{L}_{E'} &= \mathcal{L}_E + \mu\Big(\text{trace}(K(t)\bar{K}(t)) - \text{trace}(K(t)\bar{K}(t-1))\Big) \\
&= \mathcal{L}_E + \mu\Big(\text{trace}(K(t)(\bar{K}(t) - \bar{K}(t-1)))\Big) \\
&= \mathcal{L}_E + \mu\Big(\text{trace}(K(t)(\sum_{k=1}^{N} w_k K_k(t) - \sum_{k=1}^{N} w_k K_k(t-1))\Big) \\
&= \mathcal{L}_E + \mu\Big(\sum_{i=1}^{L}\sum_{j=1}^{L}(K(t)_{i,j}(\sum_{k=1}^{N} w_k K_k(t)_{i,j} - \sum_{k=1}^{N} w_k K_k(t-1)_{i,j})\Big) \\
&= \mathcal{L}_E + \mu\Big(\sum_{k=1}^{N} w_k(\sum_{i=1}^{L}\sum_{j=1}^{L} K(t)_{i,j}(K_k(t)_{i,j} - K_k(t-1)_{i,j}))\Big) \\
&= \mathcal{L}_E + \mu\Big(\sum_{k=1}^{N} w_k(\sum_{i=1}^{L}\sum_{j=1}^{L} K(t)_{i,j}(\Phi_k(x_i;\mathcal{W}_k^t).\Phi_k(x_j;\mathcal{W}_k^t) - \Phi_k(x_i;\mathcal{W}_k^{t-1}).\Phi_k(x_j;\mathcal{W}_k^{t-1})))\Big).
\end{aligned}
$$

Given the above equations, we find that

$$
\begin{aligned}
\mathcal{L}_{E'} = \mathcal{L}_E + \mu\sum_{k=1}^{N} w_k \sum_{i=1}^{L}\sum_{j=1}^{L} K(t)_{i,j}\Big(&\Phi_k(x_i;\mathcal{W}_k^t)(\Phi_k(x_j;\mathcal{W}_k^t) - \Phi_k(x_j;\mathcal{W}_k^{t-1})) \\
&+ \Phi_k(x_j;\mathcal{W}_k^{t-1})(\Phi_k(x_i;\mathcal{W}_k^t) - \Phi_k(x_i;\mathcal{W}_k^{t-1})).\Big)
\end{aligned}
$$

Taking norm on both the sides, we get

$$
\begin{aligned}
||\mathcal{L}_{E'}|| &= \Bigg|\Bigg|\mathcal{L}_E + \mu\sum_{k=1}^{N} w_k \sum_{i=1}^{L}\sum_{j=1}^{L} K(t)_{i,j}\Big(\Phi_k(x_i;\mathcal{W}_k^t)(\Phi_k(x_j;\mathcal{W}_k^t) - \Phi_k(x_j;\mathcal{W}_k^{t-1})) \\
&\quad + \Phi_k(x_j;\mathcal{W}_k^{t-1})(\Phi_k(x_i;\mathcal{W}_k^t) - \Phi_k(x_i;\mathcal{W}_k^{t-1}))\Big)\Bigg|\Bigg| \\
&\leq ||\mathcal{L}_E|| + \mu\Bigg|\Bigg|\sum_{k=1}^{N} w_k \sum_{i=1}^{L}\sum_{j=1}^{L} K(t)_{i,j}\Big(\Phi_k(x_i;\mathcal{W}_k^t)(\Phi_k(x_j;\mathcal{W}_k^t) - \Phi_k(x_j;\mathcal{W}_k^{t-1})) \\
&\quad + \Phi_k(x_j;\mathcal{W}_k^{t-1})(\Phi_k(x_i;\mathcal{W}_k^t) - \Phi_k(x_i;\mathcal{W}_k^{t-1}))\Big)\Bigg|\Bigg| \\
&\leq \mathcal{L}_E + \mu\sum_{k=1}^{N} w_k \sum_{i=1}^{L}\sum_{j=1}^{L} ||K(t)_{i,j}||\Big(||\Phi_k(x_i;\mathcal{W}_k^t)(\Phi_k(x_j;\mathcal{W}_k^t) - \Phi_k(x_j;\mathcal{W}_k^{t-1})) \\
&\quad + \Phi_k(x_j;\mathcal{W}_k^{t-1})(\Phi_k(x_i;\mathcal{W}_k^t) - \Phi_k(x_i;\mathcal{W}_k^{t-1}))||\Big) \\
&\leq \mathcal{L}_E + \mu\sum_{k=1}^{N} w_k \sum_{i=1}^{L}\sum_{j=1}^{L} ||K(t)_{i,j}||\Big(||\Phi_k(x_i;\mathcal{W}_k^t)||.||(\Phi_k(x_j;\mathcal{W}_k^t) - \Phi_k(x_j;\mathcal{W}_k^{t-1}))|| \\
&\quad + ||\Phi_k(x_j;\mathcal{W}_k^{t-1})||.||(\Phi_k(x_i;\mathcal{W}_k^t) - \Phi_k(x_i;\mathcal{W}_k^{t-1}))||\Big) \\
&\leq \mathcal{L}_E + \mu\sum_{k=1}^{N} w_k \sum_{i=1}^{L}\sum_{j=1}^{L} ||K(t)_{i,j}||\Big(||\Phi_k(x_i;\mathcal{W}_k^t)||.||\mathrm{L}_2\eta g_{t,k}|| + ||\Phi_k(x_j;\mathcal{W}_k^{t-1})||.||\mathrm{L}_2\eta g_{t,k}||\Big).
\end{aligned}
$$

Taking expectation on both sides of the above bounds, we have

$$\begin{aligned}
\mathbb{E}[\mathcal{L}_{E'}] &\leq \mathbb{E}[\mathcal{L}_E] + \mu \sum_{k=1}^{N} w_k \sum_{i=1}^{L} \sum_{j=1}^{L} ||K(t)_{i,j}|| \bigg(\mathbb{E}[||\Phi_k(x_i; \mathcal{W}_k^t)||.||\mathrm{L}_2 \eta g_{t,k}||] + \mathbb{E}[||\Phi_k(x_j; \mathcal{W}_k^{t-1})||.||\mathrm{L}_2 \eta g_{t,k}||] \bigg) \\
&\leq \mathbb{E}[\mathcal{L}_E] + \mu \sum_{k=1}^{N} w_k \sum_{i=1}^{L} \sum_{j=1}^{L} ||K(t)_{i,j}|| \bigg(R\mathrm{L}_2 \eta P + R\mathrm{L}_2 \eta P \bigg) \\
&= \mathbb{E}[\mathcal{L}_E] + 2\mu\eta\mathrm{L}_2 PR \sum_{i=1}^{L} \sum_{j=1}^{L} ||K(t)_{i,j}|| \\
&\leq \mathbb{E}[\mathcal{L}_E] + 2\mu\eta\mathrm{L}_2 PR \sum_{i=1}^{L} \sum_{j=1}^{L} R^2 \\
&\leq \mathbb{E}[\mathcal{L}_E] + 2\mu\eta\mathrm{L}_2 PR^3 L^2.
\end{aligned}$$

Thus, we have,

$$\mathbb{E}[\mathcal{L}_{E'}] \leq \mathbb{E}[\mathcal{L}_E] + 2\mu\eta\mathrm{L}_2 PR^3 L^2,$$

which concludes the proof.

B Additional Experiments

We also test our framework on the semi-supervised learning task, as described in FedU [Zhuang et al., 2021] and FedEMA [Zhuang et al., 2022] papers. Under this protocol, we let the federated SSFL methods train the local models for representation learning. For the evaluation, we assume that only a small fraction(10% or 1%) of labeled data is available. Then, after the federated representation learning of local models on clients, we fine-tune the local models along with a linear classification layer on top for 100 epochs. We evaluate our method and the baselines, FedU and FedEMA, in this setting on CIFAR10 and CIFAR100 datasets and observe that our method outperforms the baselines. For fine-tuning the models, we use SGD with Nesterov momentum as the optimizer with momentum = 0.9, learning rate = 0.05 and batch size = 512 and 256 for 10% labeled data and 1% labeled data respectively. The results reported in Table 5 and Table 6 are for federated setting with non-IID clients with 5 users, 20 classes per user for CIFAR100 and 2 classes per user for CIFAR10.

Table 5: Top-1 test accuracy for different methods under semi-supervised learning for CIFAR100.

Method	1% labeled data	10% labeled data
FedU	20.5 ± 0.5	46.3 ± 0.57
FedEMA	23.11 ± 0.65	48.6 ± 0.47
Hetero-SSFL	$\mathbf{37.4 \pm 0.95}$	$\mathbf{57.4 \pm 0.43}$

Table 6: Top-1 test accuracy for different methods under semi-supervised learning for CIFAR10.

Method	1% labeled data	10% labeled data
FedU	72.9 ± 2.25	86.8 ± 0.5
FedEMA	75.8 ± 1.4	87.4 ± 0.8
Hetero-SSFL	$\mathbf{79.3 \pm 1.85}$	$\mathbf{88.7 \pm 0.25}$